The Changes in Marine Communities Through a Discrete, Size-Structured Matrix Model

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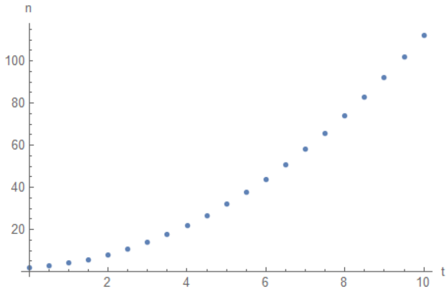
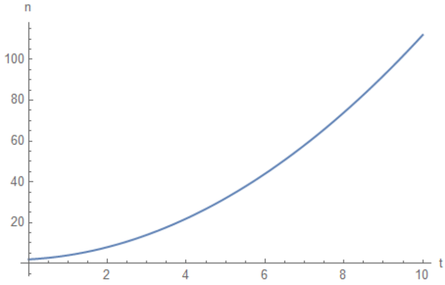
November 18, 2020

Overview

Discrete and Continuous Models

$$n(t) = t^2 + t + 2$$





$t = 0, 1/2, 1, 3/2, 2, \cdots, 10$

$t \in [0, 10]$

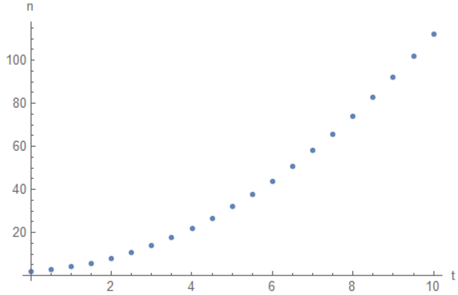
Mathematical Population Models

- We can represent population with $n(t)$.
- We can have time represented by t.
- Model could be discrete or continuous.
- Some models are determined by differential equations.

Why Use a Discrete Model?

- Some differential equations are not explicitly solvable.
- We cannot have half an individual.
- Variables in the model are easier to interpret.
- Most data collected is discrete.

Discrete Time Variable



- $t_{k+1} = t_k + \Delta t$
- For example when k = 0, $t_1 = t_0 + \Delta t$

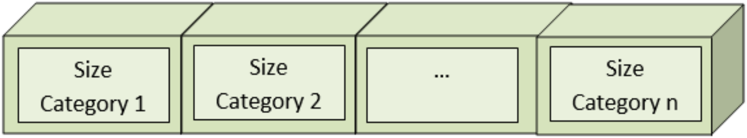
Discrete Size Classes

The mass of the *i*th size category can be represented as

$$m(i) = 0.001 \cdot (\delta)^{i-2}$$

where δ is the mass ratio between size categories.

- The interpretation of size categories and time is not the same.
- The interpretation of size is in groups.



Community matrix

$n_i(t)$ gives us the population of size category i at time t.

$$n(t) = \begin{pmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \\ n_4(t) \end{pmatrix},$$

where n_1 is the recycling size category and $n_i, i > 1$ is living beings.

Discrete, Size-Structured Matrix Model

$$n(t + \Delta t) = n(t) + \Delta n^P + \Delta n^R + \Delta n^Q + \Delta n^U,$$

where

- Δn^P is the net change in population from predation.
- Δn^R is the net change in population from reproduction.
- Δn^Q is the net change in population from metabolism.
- Δn^U is the net change in population from fishing and non-predation deaths.

Net Change in Population From Reproduction

$$\Delta n^R = \Delta n^F + \Delta n^B$$

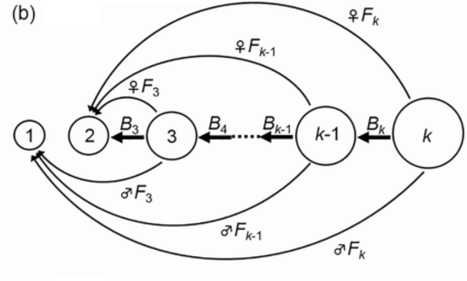


Figure: Reproduction interpretation in the model (Xia and Yamakawa, 2018).

- Δn^R is the change in population from reproduction.
- Δn^F is the increase in mass from eggs/embryos, and the increase in mass from semen and reproductive waste.
- Δn^B is the reduction of mass from birth.

Net Change in Population From The Predator-Prey Relationship

$$\Delta n^{P} = -\Delta n^{P-predation} + \Delta n^{G} + \Delta n^{E}$$

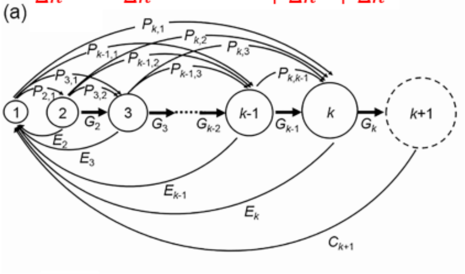


Figure: Predator-Prey relationship interpretation in the model (Xia and Yamakawa, 2018).

- Δn^{P} is the net change in population from the predation.
- $\Delta n^{P-\text{predation}}$ is the number of prey eaten.
- Δn^{G} is the number of individuals who grow.
- Δn^{E} is the number of individuals who are recycled.

Break Down of Δn^E

$$\Delta n^P = -\Delta n^{P-\text{predation}} + \Delta n^G + \Delta n^E$$

Δn^E is the number of individuals who are recycled.

$$\Delta n^E = T_1 M_1^{-1}(I - E)NVSMn\Delta t$$

Breaking Down $\Delta n^E = T_1 M_1^{-1}(I - E)NVSMn\Delta t$

$$T_1 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_1^{-1} = \begin{pmatrix} (m_1)^{-1} & 0 & 0 & 0 \\ 0 & (m_1)^{-1} & 0 & 0 \\ 0 & 0 & (m_1)^{-1} & 0 \\ 0 & 0 & 0 & (m_1)^{-1} \end{pmatrix},$$

where $m(i) = 0.001 \cdot (\delta)^{i-2}$.

Breaking Down $\Delta n^E = T_1 M_1^{-1}(I - E)NVSMn\Delta t$

$$(I - E) = \begin{pmatrix} 1 - e_1 & 0 & 0 & 0 \\ 0 & 1 - e_2 & 0 & 0 \\ 0 & 0 & 1 - e_3 & 0 \\ 0 & 0 & 0 & 1 - e_4 \end{pmatrix},$$

where $e = 0.6$ is the assimilation of predators.

$$N = \begin{pmatrix} n_1(t) & 0 & 0 & 0 \\ 0 & n_2(t) & 0 & 0 \\ 0 & 0 & n_3(t) & 0 \\ 0 & 0 & 0 & n_4(t) \end{pmatrix},$$

where n_m is the number of individuals in size category m, viewed as prey.

Breaking Down $\Delta n^E = T_1 M_1^{-1}(I - E)NVSMn\Delta t$

$$V = \begin{pmatrix} \gamma \cdot m_1^p & 0 & 0 & 0 \\ 0 & \gamma \cdot m_2^p & 0 & 0 \\ 0 & 0 & \gamma \cdot m_3^p & 0 \\ 0 & 0 & 0 & \gamma \cdot m_4^p \end{pmatrix},$$

where $\gamma = 600\text{m}^{-3}\text{yr}^{-1}\text{g}^{-p}$ is the factor for searching rate and $p = 0.75$ is the searching rate

Breaking Down $\Delta n^E = T_1 M_1^{-1}(I - E)NVSMn\Delta t$

$$S = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \phi_{2,1} & 0 & 0 & 0 \\ \phi_{3,1} & \phi_{3,2} & 0 & 0 \\ \phi_{4,1} & \phi_{4,2} & \phi_{4,3} & 0 \end{pmatrix},$$

where $\phi_{i,j}$ is a function relating predators i to their preferred prey size j.

Breaking Down $\Delta n^E = T_1 M_1^{-1}(I - E)NVSMn\Delta t$

$$M = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix},$$

where mass is calculated by $m(i) = 0.001 \cdot (\delta)^{i-2}$

$$n(t) = \begin{pmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \\ n_4(t) \end{pmatrix}$$

and $\Delta t = 0.001$ years or 8.76 hours.

Model Break Down

Discrete, size-structured matrix model:

$$n(t + \Delta t) = n(t) + \Delta n^P + \Delta n^R + \Delta n^Q + \Delta n^U,$$

Net change in population from the predator-prey relationship:

$$\Delta n^P = -\Delta n^{P-\text{predation}} + \Delta n^G + \Delta n^E$$

One of the recycling terms:

$$\Delta n^E = T_1 M_1^{-1}(I - E)NVSMn\Delta t$$

Net Change in Population Due to Metabolism

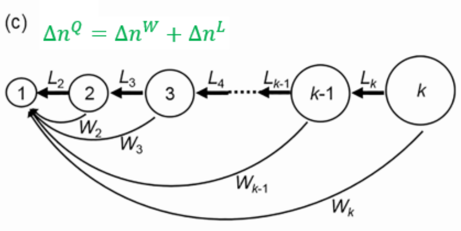


Figure: Metabolism interpretation in the model (Xia and Yamakawa, 2018).

- Δn^Q is the change in population from metabolism.
- Δn^W is metabolized mass moved to size category 1.
- Δn^L is the metabolized mass which individuals use.

Net Change in Population Due to Non-Predation and Other Mortalities

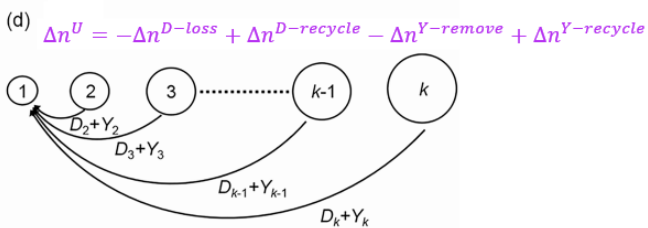


Figure: Non-Predation and other mortalities interpretation in the model (Xia and Yamakawa, 2018).

- Δn^U is the change in population from non-predation factors
- $\Delta n^{D-\text{loss}}$ is the number of deaths from non-predation factors
- $\Delta n^{D-\text{recycle}}$ is the amount of mass moved to size category 1.
- $\Delta n^{Y-\text{remove}}$ is the mass lost from fishing.
- $\Delta n^{Y-\text{recycle}}$ the mass recycled back into size category 1.

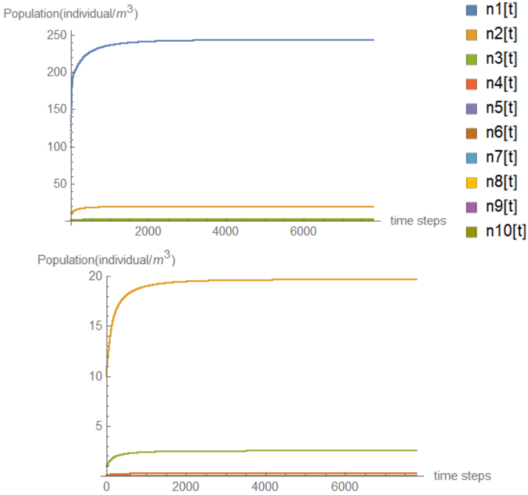
Discrete, Size-Structured Matrix Model

$$n(t+\Delta t) = n(t) + \Delta n^P + \Delta n^R + \Delta n^Q + \Delta n^U$$

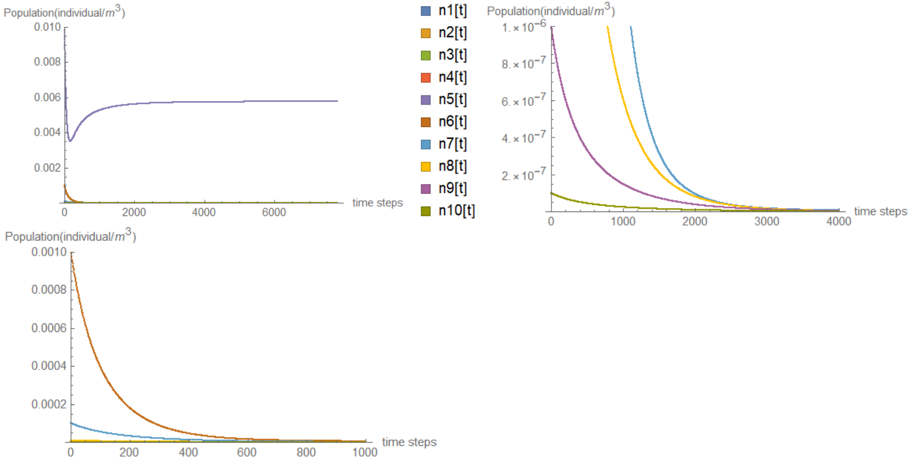
- Each net change is different.
- Conservation of mass is complicated.
- $m_1 n_1(t) + m_2 n_3(t) + m_3 n_3(t) + \cdots + m_k n_k(t)$ is constant.

Mini Model Based on the Size-Structured Matrix Model

- 10 size categories
- $\delta = 10$
- $\Delta t = 0.001$ years or 8.76 hours.



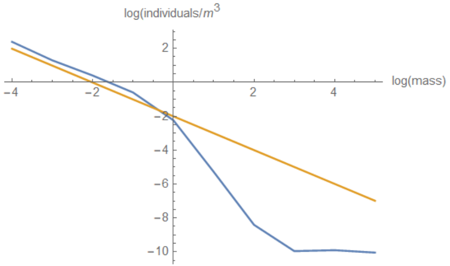
Marine Population Results with 10 Size Categories

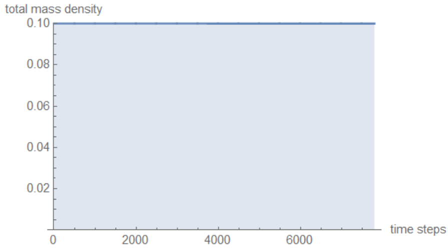


Challenges When Implementing the Mini Model

- Mass was not conserved.
- We saw extinction.
- Increasing δ allowed predators to find prey, but we could not maintain the empirical steady state.

Conclusions





- Mass was conserved.
- $m(i) = 0.001 \cdot (\delta)^{i-2}$
- $\delta = 10$ for 10 size categories.

Acknowledgement

Thank you to

- Professor Merc Chasman
- Professor Barry McQuarrie
- Family and friends

Questions?

References

Xia, S., & Yamakawa, T. A size-structured matrix model to simulate dynamics of marine community size spectrum. 2018; PLOS ONE, 13(6). Retrieved January 27, 2020, https://doi.org/10.1371/journal.pone.0198415

ϕ Fuction

$$\phi_{i,j} = \left\{ \begin{array}{ll} \exp\left(\frac{-(\log\left(\frac{\beta_i m_j}{m_i}\right)^2}{2\sigma^2}\right) & \text{if } j > 1 \\ \left(\sum_{x=-s}^{1} \exp\frac{-(\log\frac{\beta_i m_x}{m_i})^2}{2\sigma^2}\right) & \text{if } j = 1 \end{array} \right\},$$

where i represents the predator, j represents the prey, and β represents the most preferred size of prey (Xia and Yamakawa, 2018).