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Digital Computer Applications in the Natural Sciences

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A problem frequently encountered in the natural sciences is the derivation of a mathematical equation from a given set or table of numerical values. These values may be obtained empirically by the observation of some physical system in action, by the measurements of some physical object, or from hypothetical tables, obtained by means of supposition.

For purposes of illustration consider the solution of an equation involving one independent and one dependent variable of the form $Y = f(x)$. An equation with which most of us are familiar is that of

$$Y = \sin x$$

for small values of Y the curve approaches a straight line and as Y increases the curve approaches a maximum. The original plotting of the curve of sin x was done from a series of tables and the accuracy with which the curve was drawn was dependent upon the number of known points. The question is how do we determine the points that lie between the measured of hypothetically derived ones.

To obtain a curve we utilize the method of interpolation to determine the points between those plotted. Interpolation is the process by which a function, and subsequently a curve describing the function can be approximated from a table of specific points along the curve.

For instance, on a curve of sin x a linear method of approximating a point midway between two points, A and B such that $Y_A = \sin x_A$ and $Y_B = \sin x_B$, yields C′ at $Y_{C}' = (\sin x_A + \sin x_B)/2$ compared to C on the curve at $Y_C = \sin \frac{({}^xA + {}^xB)}{2}$.

The point C′ approximating C is determined by subtracting x_A from x_B and adding ½ of the resultant value to x_A.

The Y coordinates are obtained in a similar manner. $\frac{1}{2}(Y_B - Y_A) + Y_A$. This, however, assumes the curve is a straight line between the two points.

Since we cannot approximate with the degree of accuracy required using linear interpolation (unless we plot a majority of the points, which is time consuming and improbable in many instances), we must therefore, look for a method that will interpolate a non-linear function. The most useful function of this type in our mathematical knowledge is the power series.

The power series has an infinite number of terms therefore making it complex to solve. However, the straight line used in linear interpolation can be "bent" as the straight line is always of the form

$$Y = Ax + B$$

We can then replace the power series with a polynominal that accurately describes the sine function.

The mathematician, Weierstrass, presented two useful statements or theorems which are applicable to the mathematical facts of interpolation.

1. Any function that is continuous can be duplicated to within whatever degree of accuracy prescribed, by the use of a polynominal expansion.

2. If the series is periodic it may be approximated with the sine or cosine series.

It is this approach plus the use of the principles originally proposed by Charles Babbage for his "difference machine" in the early 19th Century that is used to determine the equations of the functions describing physical systems.

We may cite three examples where these routines are used:

1. Missile tracking and guidance.

From empirical information obtained from radar tracking, pertaining to velocity, acceleration, position, an equation describing the trajectory of the vehicle can be determined. This may then be compared to a previously stored equation describing the desired trajectory. If any differences exist, corrections may then be given to the missile in the form of steering orders.

2. By measuring certain points along the ocean floor and with the knowledge of the general structure of the ocean bottom, the points may be recorded and then related to give a continuous plot of ⅔ of the earth's surface.

The generation of functions is also used in design where equations predicting how a system will operate are determined from hypothetical tables obtained by methods of supposition by the designer or engineer.

This application is used in aircraft and missile design, in biomedical research in studies on blood flow and other physiological processes.

Recently new approaches have been examined in regard to wave forms. These are to determine what wave patterns can be determined in certain emissions. It is anticipated that these devices will be useful in the fields of EEG and EKG analysis.