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Geographic Potential Surfaces

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ABSTRACT—The attempt is made to trace the development of the surface potential with particular reference to the social sciences, especially geography. The geographic potential is derived from Newton's law, $F = GMm/d^2$ and is mathematically expressed and explained. The formula for the potential at a single point or area, j , is $\sum_i (P_i \cdot P_j)^k / d_{ij}^m$. Minnesota population potential for 1960 and 1930, and 1960 Minnesota income potentials have been calculated and mapped. The Twin Cities possess the peak potential for all three variables. Of prime importance to the understanding of geographic potentials is a knowledge of the underlying assumptions and limitations, of which there are many. In spite of these, the potential is of particular value in presenting an ideal model to which reality can be compared.

Three major topics are discussed in this paper. The first is the potential surface as it relates to geographic (spatial) distributions, the second is the application of computer techniques and programming to potential surface determination, and the third is a consideration of the specific application of the potential surface, including the determination and mapping of Minnesota's 1930 and 1960 population potentials, and the State's 1960 income potentials.

Potential Surfaces

The concept of the geographic potential surface is, by definition, analogous to the physical concepts of spatial potentials. Newton's law of gravitation, $F = GMm/d^2$, first described a potential area. In Newton's formula, G is the gravitational constant, M is the mass of a particle, m is the mass of another particle, and d is the distance from M to m . This formula has been applied without basic alteration to geographic distributions in determining potential values. No alteration has been needed for the shift from three dimensions to two. In 1773, Lagrange considered the attractions of several planets simultaneously by using the simple equation of mass divided by distance. These concepts were further elaborated by Laplace and Poisson. In 1828, Green applied the concept to electrical charges and termed the derived quantities "potentials"; thus, he determined the gravitational potential, the electrostatic potential, and the magnetic potential.

One of the first persons to see the analogy between physical gravitational force and social gravitational force was Henry Carey, a rather prolific writer of the mid-nineteenth century. He contended that the force in social phenomena was directly proportional to mass, population for example, and inversely proportional to distance. E. G. Ravenstein, in 1885, noted that population centers

attract migrants from other centers in direct proportion to the population, and inversely proportional to the distance between the centers. E. C. Young contended that intensity of movement between a source and terminal varied inversely with the square of the distance. James Bossard related residential propinquity to marriage selection. William Reilly, in the late 1920's, proposed the law of retail gravitation, which gave approximations to retail trade divides between two centers, using population divided by distance squared (Reilly, 1931). Although these early attempts are not very sophisticated and can be shown to be false in many specific cases, they present quantitative measurements of phenomena that would seem almost intuitively obvious. G. K. Zipf (1947) explains these measurements with the principle of "least effort." All these early studies were concerned only with specific area potentials rather than with a continuous potential surface.

Zipf (1946, 1947) and Stewart (1941, 1947, 1948) gave the major impetus to the development of the potential or gravity model. Zipf essentially continued the same type of work as Carey, Ravenstein, and others. Zipf used the population over distance factor to study the interaction between pairs of cities, considering such phenomena as bus-passenger trips, telephone calls, railway shipments, and obituary notices (Zipf, 1946). In his equation, population over distance (p/d) is raised to some power. Stewart (1948) directly applied Newton's law of gravity, changing only the gravitational constant. He came up with two abstract concepts: demographic force and demographic energy. The empirical basis for defining two concepts escapes me. By deriving each point's (area in computation) potential, a surface is described. It is a "derived quantity" that considers the distribution of people or some other geographically distributed variable. It considers both mass and distance and is really continuous. The mathematical determination of a potential surface will be defined after a brief literature survey.

William Reilly's *Law of Retail Gravitation* (1931) gives a good example of the early application of the p/d hypothesis. Reilly's equation was $P_j/d_{ji}^2 = P_i/d_{ji}^2$, that

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is, the population of city j and the population of city i , divided by the distance (squared) between each city and a certain point between the two cities, is equal. This does not fit reality in that cities do not usually interact in pairs. The equation does not take into account the qualitative differences between cities that are always present in some degree. Functions of a city are generally considered more important than size, although they are related. One other fault that has been found in Reilly's hypothesis is that the exponent of d is generally too large. Despite its numerous drawbacks, the concept of the formula has been empirically substantiated in some cases.

There are numerous works that deal with specific use of potentials. Zipf (1946, 1947) wrote a number of short articles in which he attempted to verify the p/d hypothesis. The two cited works revealed the basic underlying concepts with which he worked and, also, the limiting assumptions that he made. A third work, *Human Behavior and the Principle of Least Effort* was an example of how Zipf applied the p/d hypothesis.

Edward Ullman (1954) related potential, in general terms, to "flow." He pointed out the difficulty in measuring objectively many types of interaction. Such things as the effect of direction, transportation facilities, modes of interchange, natural barriers, and national boundaries were given as factors that distorted the correlation between population potential and flow. Ullman and Volk (1962) discussed the problem of prediction without a base. The p/d equation was somewhat naively used to predict a future reservoir's recreation use.

Duncan et al (1960) gave numerous references to population potentials, including numerous applications. They correlated potential to a number of metropolitan functions, such as percentages employed in manufacturing and in wholesaling.

Chauncy Harris (1954) derived a market potential and discussed its geographic and economic relevance with particular emphasis on the importance of market potential in industrial location. He concluded its importance by both reason and empirical findings but did not say how important. Market potential resembled population potential generally. Isard's (1960) very comprehensive study of geographic potential included a brief history of potential development in areal studies. The concept was explained with a great deal of mathematical sophistication. He gave many uses to which potentials have been applied, citing their drawbacks and also proposing some possible new applications of the established concepts. He was also quite forthright in stating what he had not discussed and cited references to these particular issues. Most were highly esoteric, such as the gradient of potential, a concept developed by Stewart and Warntz (1947).

Stewart is one of the most prolific students of geographic potentials. He first gave expression to the concept of the continuous potential surface and has written many articles describing and applying potential surfaces. In "Empirical Mathematical Rules Concerning the Distribution and Equilibrium of Population" he gave a good

history of the development of the use of potentials, the results of his findings in several of his previous studies, and by far the most lucid definition of what a potential surface represents. He applied the potential equation in the p/d form in Stewart (1941). He found that attendance, by state, at several eastern schools correlated positively to the population of the state and inversely to the state's distance from the school. In Stewart (1947) he discussed the intriguing concepts of the "human gas," and "demographic temperature." He also related population potential at various periods in history to rural density and found a positive correlation. A number of correlations between population potential and numerous socioeconomic variables were given graphically on log-log paper in Stewart (1948). Factors studied included rural density, rural nonfarm rent, farm value, railroad density, death rate, and income. Most of the correlations were quite strong, but suffered from several limiting restrictions used by the author.

William Warntz is, presently, the great advocate of the geographic potential surface. To him, it is a veritable panacea to all that ails geography; nothing is beyond its reach. He has correlated potential surfaces with everything from bank checks, flows of people and goods, to such less salient phenomena as marriage licences, business failures, taxes, mental health, and alcoholism. He has mapped U. S. annual onion-supply potential. Although he has surely gone too far in ascribing applicability to the potential, he does possess a good deal of knowledge about the mathematical intricacies of potentials and the mapping of potentials. In Warntz (1964) he presented a detailed map of U. S. population potential. Much of Warntz's work has been done in conjunction with Stewart.

Basically, the potential surface is easy to understand although most of the literature defines it poorly or just in part. The potential surface fits into "quantitative" geography as one of the many facets of surface or spatial variation. Theoretically, a potential surface is composed of the values of the potentials at all points. Since there are an infinite number of points, the process is arrived at by integrating the density d of an infinitely small area dA divided by the distance r from each point to a point c . Every point is considered in the integration process. This process gives the potential at one point. To derive the surface, all points must be considered.

The formula is: $Potential_c = \int 1/r DdA$. Simply stated, the potential at a point c is the sum of the reciprocals of the distance from c to every other point. Since this method would make the determination of a potential surface humanly impossible, a simplified computational method is used. Instead of an infinitely small area, larger, preferably equal areas, are used. The larger the area, the simpler the computation but the lower the precision. For each area, a point is chosen that is considered to be the center of mass. This mass (population, income, etc.) is multiplied by the mass of every other area and divided by the distance between the two areas. A potential value for each point (area) is the sum of the values derived by

multiplying its mass by all others and dividing by the distance between the points. The equation for one finite point, p_j , is

$$K \sum_{i=1}^n (P_j \cdot P_i)^k / d_{ji}^m,$$

where p_i takes on the values of all other points; d_{ji} is the distance between fixed point j and variable point i ; k and m are exponents with no a priori value; K is a constant that is also not given; and n is the number of points (areas). The values of k , m , and K that should be used in determining a surface qua surface would seem to be indeterminate. To the sum p_j must be added the value p_j exerts on itself. This is determined by multiplying p_j times itself (implied in the formula) and dividing by one-half the radius of the area. One-half the radius is used for want of a better measure of "distance."

What a potential is becomes quite clear in the following paraphrase of an analogy of Stewart (1947). Population potential, as an example, measures the propinquity of people. It relates the closeness of each person to every other person in a spatially continuous manner. It has been termed a measure of human influence or, more logically, accessibility. Instead of considering a person to be a fixed entity, with all his value at his location, we can consider him to possess spatial influence that decreases regularly with distance in all directions. This assumption is certainly not true for one individual but on the average it is a plausible although not proven assumption. A person is most accessible, or possesses the most influence, on the average where he is at. A regular decline of influence or accessibility with distance will give a cone of some sort, depending on the exponents of distance and mass, for each individual. The cone is spatially located and, at every point in the entire study area, is of a certain height. The total potential of a point is derived by summing the heights of all person's cones at that point. In computation, single persons are not used but, as stated before, discrete areas are used. The computational analogy uses area values instead of people. There is some question of the type of areas to use. It would seem logical to use equal areas although Carrothers (1958) has stated that the size of area should be inversely proportional to density. This may be true if the major concern is for the areas (as points) per se, rather than for the actual potential surface. If accurate surface determination is desired and the use of equal areas is not feasible, density values should be used. The potential derived will then give the potential of any unit (square mile, for example) within the area. Total area potential can be derived by multiplying the potential of one unit by the number of units in the area.

It is clear that the potential surface can be mapped and is spatially continuous as is terrain. One particular problem encountered in studying potentials is the units in which they are given. A population potential is given in people per mile; nowhere in the literature could I find exactly what this meant. It is not really too important since the potential values are really only relative figures.

The potential surface relates to geography in many

ways, as indicated in the literature survey. One of the more valuable uses of the potential is the relation of flow or circulation to potentials. Since potentials have been validly correlated to many aspects of the present circulatory system, it is not expecting too much to have a map of future potentials, which would not be too difficult to derive, to predict future flows. By use of a potential map, evaluation of an area in terms of something such as sales or attendance can be performed. To do this, the level of the activity measured must be comparable to the area mapped, that is, if a national potential map is used, the variable considered must be at a national level. It would make little sense to expect an area's national potential to be causally related to its sales of some ubiquitous product. This aspect of a potential's use has been little used and would seem to be of great future value. The use of a potential map in predicting future needs would also appear to have some practical value.

Outdoor recreation needs are one specific field in which such a map could be used. These are some of the practical and, as yet virtually untried, uses of the potential. There are, of course, a plethora of variables that potentials have been correlated to. Before they can be of much value, causal links will have to be found. These linkages are often tenuous if they exist at all. The potential surface could be applied to more than just population. Income, freight rates, and market have been used as well as onion supply. Such things as employment, various economic outputs and sales, and a host of other variables could be used as a basis for potential determination. It is even conceivable that the sociologist or psychologist could use the concept. A potential map of sociological or psychological phenomena could be of as much interest to some geographers as economic phenomena are to others. Physical quantities, for example, forested areas or lakes, could be used in determining a national recreation potential map. There are many uses that a potential can be put to and, considering the state of our society, no harm will come from any of these endeavors in terms of lost opportunities.

The Application of Computer Techniques and Programming to the Determination of Potential Surfaces

The main reason for the rapid expansion of the use of potentials in the social sciences is the advent of the computer. Hand calculation of a potential surface is so tedious and time consuming that the *raison d'être* for a potential's calculation had to be outstanding. The errors that occurred in human computation could be great. From the formula, it can be seen that with 100 points (areas) one would need to perform 10,000 (100^2) distance calculations or measurements, the same number of multiplications and divisions, and 100 summations of 100 values each. The use of the computer entails merely assigning an x and a y grid value to each of the hundred points and having them punched on cards with the value to be measured. More than one value can be punched on the card. For example, a time series of population po-

tentials or various kinds of potentials could all be performed in one computer run with only minor program alterations. The potential values derived from each variable could be stored and, with an addition to the program, could be correlated. Identifiers for each point can be used if desired by simply punching the names on the respective cards. The only other necessary information, besides the variable values and the x and y values, is one-half the radius values. The last three must be in the same units—logically miles. All that is needed is 100 cards. The Univac 880 computer can normally handle a one-variable, 100-value-potential problem programmed in Fortran in less than 1½ hours. The program, as compiled by the computer, is given with an explanation in the appendix.

Minnesota Population and Income Potentials

Three Minnesota potential surfaces have been mapped. The first map, Minnesota population potential for 1960, shows the dominance of the Twin Cities. Southern Minnesota and the Duluth-Iron Range area are also quite strong. The pattern is one of concentric rings about the Twin Cities, showing a marked deviation toward the south. Northern Minnesota, except for the Duluth area, and Western Minnesota, except for Clay County, are weak. Wabasha County in the southeast is an isolated low spot. Pine and Aitkin Counties are also lower than their relative positions would suggest. It is clear from the map that potential does relate population to distance. The 1960 income potential map resembles the population potential to some extent although the income poten-

tial is much more concentrated in the Twin City area. Income potential is also concentrated in the south, particularly around Rochester, and also around Duluth in

MINNESOTA INCOME POTENTIAL
1960

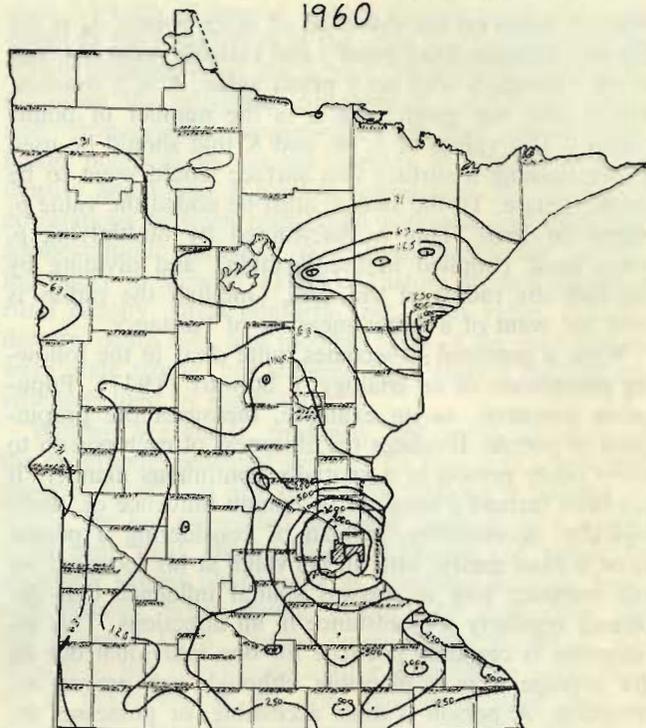


FIGURE b.

MINNESOTA POPULATION POTENTIAL
1960

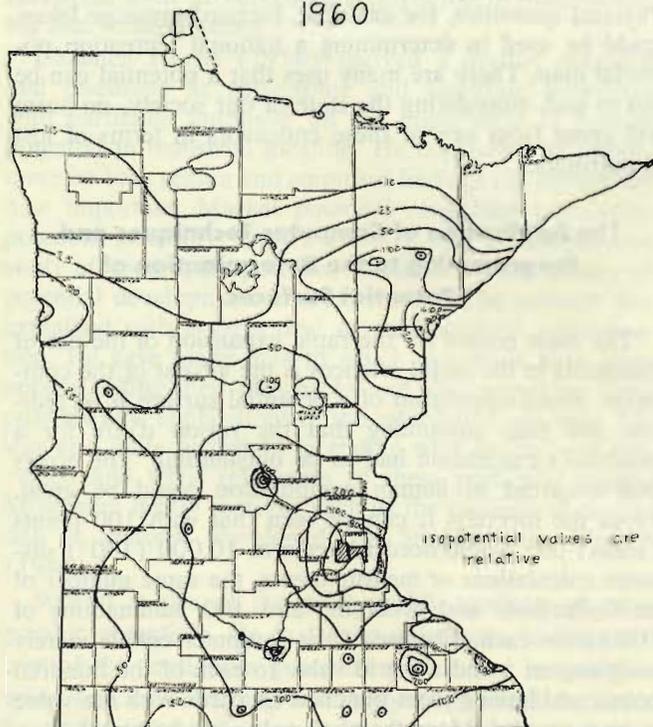


FIGURE a.

MINNESOTA POPULATION POTENTIAL
1930



FIGURE c.

the north. Population potential for 1930 correlates much more strongly to 1960 population potential than did income potential. There has been an increasing concentration of potential around the Twin Cities since 1930.

Population distribution is roughly similar to income and population potential. This is certainly to be expected since the distribution of population does, in a sense, define population potential. Population change is less correlated than is population distribution. Other distributions and patterns that correlate to Minnesota population potential include, growth in convenience centers and hamlets, the highway network, and the Minnesota dairy farming-region.

Conclusions

This paper gave only a cursory study of the factors relating to the various potential surfaces. The basic limiting assumptions used in potential determination should be emphasized. All the assumptions are false for the individual but are considered to be true on the average. Even on the average it is doubtful if these assumptions would hold up for a very large area. The first major assumption is that all people, dollars, or whatever is being measured have the same influence or accessibility. Equality of humans may be morally justified but it is not true economically, politically, or socially.

A second major assumption is that influence or accessibility declines regularly with distance, the same amount in all directions. This would be extremely difficult to measure even for one individual, and is even more difficult to speculate about in aggregate terms. If influence is related to agglomeration or to regional location, or if there are regular deviations in directions of influence, then the potential map will be incorrectly biased. Until such deviations are measured, we have no alternative but to use the present assumptions. Stewart (1949), in his study of eastern schools, did in fact use weighted values for certain people to make his equations conform with reality. Negroes were given a value of $\frac{1}{3}$ and westerners a value of 2.

A third assumption is one usually made by default. Since no a priori reasons for choosing an exponent of distance or mass exist, both are usually left at unity or distance may be squared. When specific pairs are studied, following Zipf, exponents could be determined if we could objectively measure influence or accessibility over distance. Only when we try to correlate potential surfaces to other distributions do exponents really make sense. They are usually determined arbitrarily or by trial and error.

The fourth and final assumption is that the region being studied is not influenced from outside. This is often not a valid assumption for any region except the world. One way to overcome this difficulty is to add certain neighboring areas in calculating the regional potentials. The problem of when to stop adding neighboring areas comes up when this method is used. International boundaries and marine distances present extremely difficult theoretical problems. Until better estimates of external

influences are available, we are forced to use the present straightforward method.

In spite of the many drawbacks, potentials are particularly valuable in presenting an ideal model to which reality can be contrasted. In this way, the direction and magnitude of the deviation can be measured, and it is the deviations and their causes that are of most concern.

Several applications of the potential surface to geographic study have already been mentioned. One of the big needs in the field of potential surface study is a discussion in laymen's language. Stewart has come closest to this. Before the potential concept can be of any value, it must be clearly understood by many more people. A second field of study should be concerned with systematically studying the assumptions, limits, drawbacks, and logical applicability of potentials. This is especially crucial where such sophisticated mathematical and physical concepts are involved. With the advances of computer techniques, we should be able to derive much more accurate potential surfaces.

Correlations should be made, as in the past, between various distributions and the more accurate recent potential surfaces. The process should not end there. It should be determined whether or not there are other distributions that correlate to a certain variable more strongly than the potential values do. Causality should be studied also. It must be remembered that correlation does not mean causation. This type of study will necessarily be very difficult but should be attempted.

As suggested before, there are a multitude of specific applications to which the potential could be applied, but it is my opinion that before potentials are applied to every phenomenon under the sun, the methods, basic concepts, and limits should be more precisely defined and more widely known.

Appendix

The calculations used in drawing the four potential maps of Minnesota were determined using essentially the same program, written in Fortran, a symbolic computer language. To use a computer for a mathematical problem of this type, it is necessary to translate the formula into a language the computer can understand. Fortran is such a language. The computer has no tolerance of mechanical errors. It will print out whatever error has been committed. Logical errors are not listed but will either stop the program, send the computer into a never ending cycle, or print out the "wrong" answers.

Table 1. Compilation of Computer Program.

```

1 C DEMOGRAPHIC POTENTIAL SURFACE PRO-
   PROGRAM IN FORTRAN. M. MUNSON
2 C
3 DIMENSION A(100), B(100), C(100), R(100),
   X(100), Y(100), Z(100)
4 N = 87
5 DO 1 = 1, N
6 1 READ 2, A, B, C, R, X, Y, Z
7 2 FORMAT (A5, A5, A5, F8.0, 37X, 3F4.0)
8 DO 6 K = 1, N

```

```

9     POT = 0.0.
10    DO 5 L = 1, N
11    SUM = RK·RL
12    IF(L - K) 3, 4, 3
13    3 DIST = SQRTF (XK - XL) · (XK - XL)
      + (YK - YL) · (YK - YL)
14    GO TO 5
15    4 DIST = ZK
16    5 POT = POT + SUM/DIST
17    6 PRINT 7, AK, BK, CK, RK, POT
18    70 FORMAT (3A5, 26H has a 1960 POPULATION OF,
      F11.0, 31H AND A
19    1 POPULATION POTENTIAL OF, F25.2,/)
20    STOP
21    END

```

The program used in computing the potentials is quite short and is given here. Line 1 is a comment, naming the program. Line 2 merely skips a line. Line 3 tells the computer to set aside 100 units of storage for each of the variables. *A*, *B*, and *C* are identifiers; *R* is the value used in computation; *X* and *Y* are grid values; and *Z* is ½ the radius of each area. Since the computer sets aside 100 units of storage, no more than 100 values for any variable may be used. Line 4 tells the computer that there will be 87 values. This number must exactly coincide with the number of variables. Line 5 is a "do statement" that will have the computer perform what is below it, up to and including statement 1 (from Do 1), 1 to N(87) times. Line 6 tells the computer what to "do," that is, read in variables *A*, *B*, etc., from all 87 cards. Line 7 tells the computer the location and form of the data on the cards. The values used in computation in this program (*R*, *X*, *Y*, and *Z*) must have decimals. The format statement can be varied to read in different variables for computation. Lines 8 through 16 contain the statements that instruct the computer in calculating the potentials. Line 8 will assign the values 1 through N, to K. K will change every time the computer reaches statement 6 (line 17). It is a cyclical process that is repeated 87 times. Line 9 zeroes the potential value each time it has been printed, so that it does not accumulate. Line 10 is another cyclical "do statement." It will assign to L the values 1 to N for every value of K. That is, it takes on the values 1 to 87, 87 times. This cyclical occurrence goes to statement 5. The two "do statements" give L and K subscript values that are assigned to R. In line 11, R_K will be multiplied by R_L. Since for each value K, L varies from 1 to 87, the sum of each area value times every other is obtained. Due to the arrangement of the termination and beginning of the "do statements," each potential value will be printed as it is calculated. Line 12 is an "if" statement that determines, by subscript analysis, whether the value of *F* is being multiplied by another value of *R* or by itself. If we have, for example, R₁₀ times R₁₀ we cannot use a distance between, but we must use a Z value. Line 13 is the distance formula $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ in Fortran. It is used when R_K ≠ R_L. Line 14 has a command to skip to statement 5, avoiding statement 4, which is used only when R_K = R_L. Line 15

gives the Z value, Z_K. Line 16 sums the potential values by accumulating SUM (line 11) divided by DIST (distance, line 13 or 15, whichever is applicable), 87 times. That is, L varies from 1 to 87. Line 17 is reached when L = 87, and then the identifiers, value of R, and the potential are printed. After one value is printed the process returns to line 8. K is incremented by one, and this process continues until K equals 87. Line 18 tells the computer how to print the variables. It will print out, besides the variables, a statement identifying what has been determined. One line of printed output would look like this:

CROW WING HAS A 1960 POPULATION OF 32134.
AND A POPULATION POTENTIAL OF
1132504900.00

The identifying statements must be previously set up. Line 19 is a continuation of line 18 and lines 20 and 21 stop the computer. To explain the problem clearly and concisely is rather difficult, but it is hoped that some insight into programming has been gained by the reader.

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Book Notes

Balsam Fir: A Monographic Review, by **Egolfs V. Bakuzis and Henry L. Hansen**. University of Minnesota Press, Minneapolis.

An exhaustive survey of the literature on the balsam fir, providing a coherent picture of the species and its place in nature and forestry practice.

In the search of the literature, over 2000 sources were consulted and considerably more than half of them are cited in the book. The references are organized in an ecological framework and cover the period from the seventeenth century to the present. The balsam fir is used extensively in the pulp and paper industry, and is known to millions as a traditional Christmas tree. In North America it is a major tree species in Canada, in the northeastern United States, and in the Great Lakes region.

The book contains the following chapters: Botanical Foundations, Geography and Synecology, Ecological Factors, Microbiology, Entomology, Reproduction, Stand Development, Growth and Yield, and Utilization. Appendixes list fungi and myxomycetes and insects associated with balsam fir. There are 30 illustrations, including a frontispiece drawing by the noted nature artist, Francis Lee Jaques. Some of the individual chapters were written by co-authors.

The senior authors are faculty members of the school of Forestry at the University of Minnesota.

From Atoms to Infinity, by **Clifford D. Simak**. Harper & Row, New York.

The author, science editor of the *Minneapolis Tribune*, has based the book on articles appearing in his Monday morning science series. Nine scientific areas are covered by 14 of the country's great scientists and scientific writers.